Business and Social
Sciences Proceedings

2026 3rd International Conference on Business Economics, Education, Arts and Social Sciences(EASS & APMS 2026)

Article

Mapping Triadic Transformations: Computational Parsing and Visualization

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Abstract: In this paper, a framework for the analysis of chord progressions based on the neo-Riemannian music theory is introduced, focusing on the Parallel (P), Leittonwechsel (L), and Relative (R) transformations. This paper is a continuation of the theoretical framework of music analysis, particularly the geometric and algebraic approaches of Fiore et al. A parser is introduced that can analyze raw symbolic musical data into normalized pitch class sets. These sets can then be transformed into geometric data for the analysis of the relationships of the chords. The framework is useful in analyzing conventional and non-traditional chord progressions, such as those produced by computers, which do not follow the conventional diatonic norms. The system can transform the raw symbolic data into two- and three-dimensional tonal space diagrams that can illustrate the relationships of the chords, which may not be clear in the conventional symbolic notation. The results show the potential of the use of computers in expanding the conventional models of music analysis.

Keywords: Neo-Riemannian theory; PLR transformations; Tonnetz; Computational music analysis; Chord parser; Visualization; Harmonic cycles

Received: 26 January 2026
Revised: 08 March 2026
Accepted: 23 March 2026
Published: 26 March 2026

1. Introduction

The study of chord transformations and their structural representation has long been a central topic in music theory, particularly within the framework of neo-Riemannian analysis. Foundational research established the principles of the Parallel (P), Leittonwechsel (L), and Relative (R) transformations as core operations within a transformational network of triads. The Tonnetz model provides a comprehensive framework for representing these relationships through both geometric and algebraic structures. In particular, conceptualising the Tonnetz as a two-dimensional lattice of pitch-class space enables the systematic analysis of PLR transformations between major and minor triads. Such geometric representations have proven valuable in both theoretical inquiry and computational applications.

Earlier studies have largely focused on idealised or carefully composed chord progressions, where transformational sequences follow coherent and aesthetically consistent patterns [1]. However, real-world and computer-generated musical data often deviate from these ideal conditions, producing irregular or "noisy" progressions. This creates a need for analytical approaches capable of identifying underlying structure within such variability. A key challenge lies in developing computational models that can analyse chord progressions, map them onto geometric tonal spaces, and recognise transformation possibilities while preserving the interpretive strengths of the Tonnetz framework.

Building on this theoretical foundation, the present study introduces a chord parser designed to analyse sequences of triadic chords from both structured and unstructured datasets. The parser is capable of processing symbolic representations such as MusicXML, enabling precise identification of pitch content and its subsequent mapping within tonal space. By integrating mathematical music theory with computational methods, this approach facilitates both qualitative and quantitative exploration of chordal relationships [2].

The visualisations developed in this study demonstrate the potential of two- and three-dimensional tonal space representations not only to validate established or hypothesised relationships-such as hexatonic cycles and Weitzmann regions-but also to reveal connections that may not be readily apparent through purely symbolic analysis.

2. Problem Formulation

The basic problem that this research seeks to address concerns the means by which raw symbolic musical data, in this case, sequences of triadic chords, can be transformed into a form that can be analyzed in a systematic fashion. Although the theoretical foundations upon which such data can be transformed are well-supported in neo-Riemannian music theory, there exist several problems that must be overcome in the actual process of parsing arbitrary chord sequences and representing them in some form that can be analyzed visually and computationally.

In this specific case, the problem that was identified had to do with the fact that we wished to analyze not only musically sensible and coherent chord progressions, but also intentionally dissonant and irregular ones, such as computer-generated "random" chord progressions that do not adhere to conventional voice leading norms. Such chord progressions often exhibit sudden intervallic changes, unusual chromatic motion, and non-diatonic motion, which can greatly obscure the underlying transformational patterns [3].

To the end, we have implemented a computer parser that can read in sequences as represented as lists of MIDI note numbers [4]. This format was chosen for its precision in encoding pitch information and its independence from enharmonic spelling (e.g., C♯ vs. D♭), allowing for unambiguous pitch-class extraction. The parser normalizes chord representations, reduces them to pitch-class sets, and organizes them into a data structure suitable for geometric mapping in either a 2D or a 3D space [5]. By structuring the problem in computational terms, we created a workflow that moves directly from symbolic input to analytical visualization, preserving both the theoretical rigor of transformational analysis while remaining flexible enough to accommodate unconventional or algorithmic harmonic sequences. This enables both structural recognition and exploratory analysis in contexts where traditional manual methods may be impractical or fail to detect patterns embedded within irregular progressions [6].

3. Preparation

In order to analyze the music XML file, a parser is designed to handle the list of pitch values that are grouped into triads. This allows the data to be prepared for analysis or visualization. The main concept is that the triad is a list containing three numerical values representing the pitches. These values are the MIDI note numbers. They are contained within a larger Python list that is a sequence of musical events. The parser processes these events according to the design. At the heart of the parser is a loop that processes the list of triads. For each triad contained within the list, the parser checks if the triad contains exactly three values. This is a crucial part of the design as it helps to filter out any incomplete data that might cause errors. Once the data is confirmed to be a triad, the values are unpacked into three different variables representing the pitches. These are usually named something like note1, note2, and note3, representing the vertical distribution of the pitches within the triad. These values can be mapped directly to numerical coordinates within a two-dimensional or three-dimensional space.

Table 1 is a sample pseudocode that illustrates the chord extraction. In the loop, the chord variables are lists that contain the triads. The if statement checks if the triad is complete. This is an example of defensive programming that is crucial when working with musical data. In the real world, musical data is not always perfect and can contain errors like missing notes or extra data. By ensuring that the data is a strict triad, the parser is able to guarantee consistency and reliability (As shown in Table 1).

Table 1. Extraction Logic.

```
procedure EXTRACTION(triads)                ▷ Extracts the notes from the chord
    $chord = triads[3]$
    if len(chord) == 3 then                 ▷ COMMENT
        $chordN[0] \leftarrow note1$
        $chordN[1] \leftarrow note2$
        $chordN[2] \leftarrow note3$

    return b                                ▷ The gcd is b
```

Once the pitches have been unpacked, the parser can apply optional transformations or scaling operations to map the pitch values to a target coordinate space. For instance, the x-axis could be mapped to the first pitch, the y-axis to the second pitch, and the z-axis to the third pitch. This configuration maintains the relative pitch structure for each chord and enables a visual representation of harmonic relationships. Although the parser itself is not specific to any particular visualization format, its output is formatted in a manner that facilitates easy integration with visualization tools.

To ensure readability and flexibility, the parser does not hard-code any limits on the range of pitch values. This configuration enables the parser to accommodate both standard and extended-range instrument configurations without modification. Furthermore, the modular configuration of the parser makes it simple to adapt to datasets that include chords consisting of more than three notes by adjusting the unpacking logic.

Another important aspect is that the parser can store the processed data in a new list or write it to a file for future use. For instance, after extracting the pitches, the parser can store the extracted values in a list of coordinate tuples:

This configuration is compatible with popular visualization tools like Matplotlib and Seaborn, which often require data in tuple or array format.

In summary, the parser provides a clear and consistent method for extracting and preparing the triadic chord data from the raw data set. By following good programming principles that are both simple and effective, the parser ensures that the output is easily integrated into any form of musical analysis or visualization that might be required.

4. Beethoven Example: 3D Visualization Analysis

4.1. Spatial Distribution In Pitch-Class Space

In analyzing the 3D visualization of the Beethoven progression under analysis, it is immediately clear that the spatial distribution of the chords within the pitch-class space is not random. As the tonal nodes are plotted on the surface of a lattice representation of the harmonic space, it is clear that the distribution of the triadic harmony is not random. One of the more interesting configurations is the presence of several "3-1" groupings that are evident throughout the data set. These are clusters that contain three chords that form the vertices of a small obtuse triangle within the space, with a fourth chord that is connected to the group. Unlike the geometric shapes that would be evident if the data were arranged in a more Euclidean space, the data is arranged within a pitch-class topology that is based on the neo-Riemannian transformation. As a result, the "distance" between the shapes is based on the minimal voice-leading displacement.

4.2. Edge-Connected Visualization

When the edges connecting the nodes are included in the visualization, the structural clarity is greatly enhanced. The term "3-1" clusters is seen to resolve into a group of tightly interconnected triplets that share one or two types of edges, while the fourth and outlying chord is seen to be connected by a single transformational pathway. The visualization enables one to follow the exact progressions by tracing a path through the network. For instance, two of the triangles in the group can be seen to belong to what can be identified as a segment of a hexatonic cycle, with P-L alternations tracing a single triad through a cycle of six harmonies. The structural similarity to the structural similarity to the opening of Beethoven's Op. 109 is particularly noteworthy: in each work, there can be seen to be moments of harmonic development that are governed less by diatonic-functional logic than by the symmetries of the transformational space itself.

4.3. Edge-Free Visualization

However, if the same three-dimensional structure of chords is visualized without the edges, the interpretation is completely different. In the structure without edges, the viewer is not aided in any way to understand the relationships that the chords have with one another, and the only way that the viewer is able to understand the relationships is based on the way that the points are visualized in space and the way that the viewer remembers the relationships that were visually represented. In the structure without edges, the repetition of the pattern "3-1" is more visible. Three chords form an obtuse triangle, and one is placed away from the other three. This pattern is repeated in several parts of the structure. In this structure, the way that the chords are close to one another is based on the way that they are visualized in space, not necessarily the way that they are close in terms of musical relationships. There are chords that are very close to one another but do not have strong relationships, and there are chords that have strong relationships but are not close to one another. The removal of the edges also impacts the total visual appearance of the structure. It is no longer as network-like and more resembles a series of clusters separated by space. The clusters consist of these small obtuse triangles and points. They create a visual motif that is repeated in the space. These repeating visual motifs can be considered local harmonic neighborhoods where chords have similar content or properties. They may not necessarily be related to each other in the original network. This way of visualizing the network is more focused on the spatial groupings of the chords rather than the transformational relationships between the chords. It shows that the same harmonic relationships can be considered from different points of view depending on how the information is presented. It is a less visually complex form that has the same level of complexity. It shows the potential transformational paths that the analyst might consider. It also shows patterns that could be missed if the network is considered as a whole.

4.4. Hexatonic Cycles In The Beethoven Progression

In the Beethoven progression under consideration, various types of partial and complete hexatonic cycles can be found. The term "hexatonic cycle" used in this context refers to the set of six triads resulting from the sequence of Parallel (P) and Leading-tone exchange (L) transformations. The triads of the hexatonic cycle contain two common tones with the two adjacent triads, resulting in minimal voice leading displacement of one semitone between consecutive triads. On the spatial visualization, the cycles take the form of nearly planar hexagons located in the 3D lattice, sometimes with a certain angle with respect to the Cartesian coordinate system, due to the non-Euclidean geometry of the space. However, not all of the hexagons in the progression represent complete hexatonic cycles, as some of the cycles are truncated, with only three or four triads comprising the cycles, after which the sequence proceeds differently in accordance with the next type of transformation. However, the truncated cycles should not be ignored, as they possess the characteristics of the complete hexatonic cycle, thus serving as modular harmonic units in some other contexts, often tonality-oriented.

4.5. Weitzmann Regions And Their Transformational Role

In relation to the hexatonic cycles is the Weitzmann region. This is composed of three triads that form an augmented triad. The Weitzmann region is bounded by two Parallel transformations and one Relative transformation. This indicates that the Tonnetz is composed of a triangular structure. In the Beethoven progression, a number of local transformations take place within one Weitzmann region before moving out to other regions. This local movement within the Tonnetz is akin to a harmonic orbit. This is because a local movement occurs within a limited number of transformations before moving out to a different region through an edge. In terms of the spatial configuration within the 3D Tonnetz visualization, the Weitzmann region is seen as an equilateral triangle. These equilateral triangles are also seen as part of a hexagonal structure as previously discussed. The relationship between the hexatonic cycles and the Weitzmann region indicates that a dual transformational system is at work within the Beethoven progression. This is because a dual interaction is seen between the hexatonic cycle and the Weitzmann region. This is because the hexatonic cycle is seen as a long-range cyclic motion, while the Weitzmann region is seen as a local movement where two tones remain stationary and one tone moves up a semitone (As shown in Figure 1).

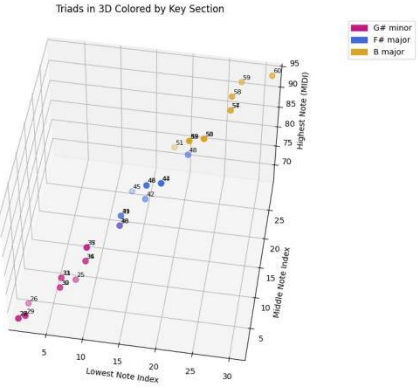

Figure 1. 3-D visualization with no lines.

In sum, the 3-D visualization of the Beethoven passage, with and without edges, represents a rich medium through which harmonic structure can be analyzed. The version with edges, in particular, emphasizes the functional connectedness of the chords, in which hexatonic cycles and Weitzmann regions can be seen as clearly demarcated groups. The version without edges, on the other hand, emphasizes the spatial clustering of harmonies, in which analysis in terms of proximity and density can be seen as relevant. Collectively, these versions suggest that the harmonic structure represents a space in which Beethoven's choices can be seen not in terms of surface-level chromaticism, but as a series of navigations through a complex, symmetrical space that is equally accessible from the point of view of music theorists emphasizing transformation and those seeking broader aesthetic connections with his late-period works.

4.6. Three-Dimensional Visualization of the Simple PLR Sequence

In the three-dimensional visualization of the simple PLR sequence, each chord is depicted as a separate node in a coordinate space, with the axes representing different pitch classes translated into integers within a defined range. This enables the viewer to observe the progression of harmonic content within the space, exposing the intrinsic topology generated by the transformational process. In the case of this example, we start with a triad placed within the interior of the defined middle area and use a controlled sequence of P, L, and R transformations. This, according to neo-Riemannian theory, moves the triad along a network of related harmonies, tracing a continuous path within the three-dimensional harmonic space. The P transformation, or Parallel, maintains two common tones and shifts the third pitch by a semitone to toggle between major and minor harmony types. The L transformation maintains two common tones and changes the third pitch to produce a semitone voice leading in the opposite direction. The R transformation, or Relative, maintains two common tones and moves the third pitch by a whole step, moving between relative major and minor keys.

4.7. Successive Transformations And Pathways

If the transformations are applied successively, the resulting pathway is a smooth or almost smooth ribbon-like trajectory through the harmonic space. The pathway is not arbitrary but is determined by the stringent constraints of the PLR operations that keep the chords closely related in the neo-Riemannian network. As the pathway progresses, groups of nodes appear that correspond to the repeated back-and-forth transformations of similar harmonic regions. In the 3-D space, these groups appear as compact formations or blocks of nodes, while the connecting edges between these blocks appear as bridges or connecting lines between the discrete harmonic zones.

4.8. Voice-Leading Economy And Geometric Forms

The form of the visualization is also influenced by the voice-leading economy of the transformations. The fact that each of the PLR transformations involves the displacement of only one pitch by a small interval means that the nodes of the successive chords appear close together in the coordinate system. The edges between the nodes appear short and straight lines that together create pathways that highlight the underlying transformational logic of the system. In areas of the system where the transformations are repeated cyclically-such as the P-L or P-R transformations-the resulting geometry reveals loops that are a characteristic feature of hexatonic cycles in the neo-Riemannian system of analysis. In the hexatonic cycles, the pathway is a symmetrical polygonal structure in 3-D space. The third dimension changes the appearance of the Tonnetz in comparison to the 2-D version, but the cyclic order of the chords is still discernible.

4.9. Effect of Removing Connecting Edges

The removal of the connecting edges in the visualization changes the point of focus from the transformational pathway to the positional arrangement of the chords themselves. This removal of edges enables the viewer to focus on the clustering and density characteristics of the chords, rather than their sequential movements. This allows for the identification of regions where the harmonies tend to congregate due to their shared pitch characteristics. As observed in the simple PLR transformation, the removal of edges indicates that the chords do not uniformly disperse across the entire possible pitch region, but instead tend to congregate in a small portion of that region due to their constrained nature.

4.10. Analytical Value of the Visualization

From an analytical perspective, the importance of this visualization resides in the manner in which these relationships are made concrete. The occurrence of cyclic geometric forms, especially those resulting from hexatonic cycles, emphasizes the underlying structural coherence of harmonic movement within the transformational system. In neo-Riemannian theory, a hexatonic cycle results from the repeated alternation of two of the three basic transformations, which results in a closed set of six triads that are maximally related through efficient voice leading. For example, the repeated alternation of P and L transformations will yield a Weitzmann region, which consists of three major triads and three minor triads that collectively form a symmetrical hexagonal pattern within the Tonnetz structure. In the 3-D visualization, the Weitzmann region assumes the form of a compact repeating geometric pattern, where the edges of the pattern correspond to minimal voice leading changes. The same is also true for the P and R transformations, although their spatial configuration will differ due to the whole-step displacement in the R transformation.

4.11. Conclusions From the Simple PLR Sample

From this simple PLR sample, it is clear that there is a compelling model to be used to illustrate the effect of transformational constraints on the creation of geometric patterns in higher-dimensional visualizations. The pattern of edges between the nodes illustrates the specific transformational history, and the absence of edges between the nodes illustrates the static pattern of harmonies. An appreciation for the combination of the two perspectives on the progression, with and without the edges, is necessary to gain a deeper appreciation for the voice leading and how it impacts the harmonic progression. The repetition of the hexatonic cycle and the Weitzmann regions in the short, artificially generated sequence of events lends credence to the idea that these patterns are not anomalies but, rather, the natural result of the transformation constraints. It is clear, then, that the neo-Riemannian operations are not only valuable in that they provide compelling theory to explain the patterns of harmonic proximity but that the operations themselves are compelling patterns of geometric repetition. As such, the 3-D visualization is not simply the visualization of an abstract mathematical operation but, rather, the direct representation of the harmonic logic inherent in the operation.

4.12. Randomized Results

The 3-D visualization of the "random noise" music composition has revealed that the tone landscape is quite fragmented and random, with little resemblance to the patterns that can be observed in more structured music compositions. Each chord in the music composition, marked by a point in the 3-D tonal space, is located at what appears to be arbitrary coordinates in the 3-D tonal space. The distance between successive points varies wildly, and there is little indication that the points might be arranged in a linear fashion or that they might form a geometric pattern. The arrangement of the points appears to be arbitrary and random, and some points are close together by chance, whereas others are far apart. There is little indication of voice leading patterns, and the observer cannot readily follow a pattern that would represent the minimum distance between successive chords.

When edges representing voice-leading connections are drawn, the picture remains chaotic. Some edges stretch long distances across the space, cutting through regions without passing near other points. Others connect chords that are close in space but bear little harmonic resemblance. There are occasional tiny clusters-two or three points lying close enough to suggest a brief moment of local coherence-but these are quickly abandoned for wide leaps into new, unconnected regions. This constant alternation between local density and empty space produces a jagged network of connections, lacking the regularity or symmetry required for interpretable transformational analysis.

Viewing only the chord positions further emphasizes the randomness of the structure. Unlike in a well-organized tonal graph, where one might identify recognizable shapes such as compact triangles or lattice-like arrangements, here the chords form an irregular scatter. The eye finds no repeating pattern or obvious grouping beyond chance proximities. An example of such a visualization is given in Table 2.

Table 2. Chord triplets from the randomized sequence, visualized as raw MIDI values.

[67, 71, 102]	[69, 80, 92]	[61, 64, 88]
[85, 94, 107]	[34, 35, 39]	[89, 101, 108]

[34, 36, 40]	[83, 91, 106]	[85, 90, 102]
[99, 101, 106]	[73, 74, 95]	[56, 57, 95]
[54, 58, 80]	[31, 43, 77]	[82, 91, 106]
[99, 105, 108]	[48, 60, 96]	[105, 107, 108]
[72, 73, 106]	[80, 81, 92]	[60, 65, 96]
[94, 100, 101]	[102, 107, 108]	[99, 103, 108]
[39, 50, 81]	[72, 84, 89]	[61, 63, 69]
[46, 49, 78]	[60, 70, 96]	[101, 102, 103]
[44, 45, 67]	[81, 90, 97]	[75, 82, 86]
[21, 22, 58]	[40, 48, 61]	[46, 50, 91]

Because the data does not conform to tonal organization, the mapping between visual proximity and harmonic relationship is almost entirely broken. Two chords that happen to be near each other in Euclidean space may be harmonically distant, and vice versa. The tonal space here resembles a disordered lattice, akin to a crystalline structure with displaced atomic positions from their ideal positions, producing irregular voids and occasional accidental overlaps. There is no clear notion of harmonic "regions" such as those that arise in structured music; instead, the observer encounters a tonal wilderness, where each new chord represents a jump to a fresh, unrelated location.

From an analytical perspective, this 3-D scatter makes the observer's task qualitatively different from working with a structured progression. Rather than following a guided path through connected harmonic neighborhoods, the viewer must interpret each point in isolation, occasionally noting small coincidences of proximity but never being able to predict the next location (As shown in Figure 2).

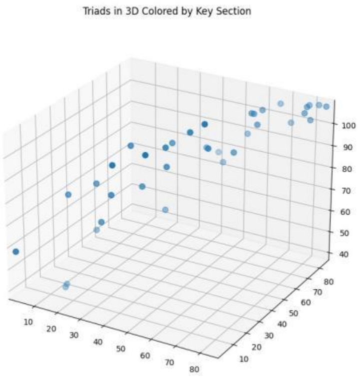

Figure 2. 3D scatterplot of randomized chords without transformational edges.

This form of visualization also illustrates the point that tonal space is dependent on the intent of the composer. Without the intent, the space is a jumbled mess where the location of the chords means nothing. The resulting tonal space is fractured and disjointed, a random assortment of points without the organization normally present in a harmonically arranged set of chords.

This study created and tested a computer parser to analyze chord progressions using the neo-Riemannian theory. The parser was able to take the symbolic musical input, normalize the pitch class content, and convert the chords into geometric shapes to perform the P, L, and R transformations. Both constructed and randomly generated chord progressions were analyzed, with an emphasis on the simple PLR cycles and their presence in the Tonnetz space. The results of the analysis indicated that some of the chord progressions, even those that are not diatonic, can be seen as clear geometric shapes, such as the small obtuse triangle that consistently appears in the simple PLR case.

This study demonstrates the effectiveness of using the algorithmic parsing and geometric shapes to illustrate the movement between chords in the harmonic space. The computer parser assists the theory as well as the visualization of the theory by providing a visual representation of the input. In the example provided, the Beethoven piece, the randomly generated chord progression, and the simple PLR case all illustrate the effectiveness of the parser in handling the various harmonic contexts while providing useful geometric shapes.

However, there are some questions that remain to be answered. First, the parser was able to effectively analyze the Beethoven piece, the randomly generated chord progression, and the simple PLR case, but it is unclear how the parser would perform with more complex or mixed systems of transformation. Second, the addition of rhythm, voice leading, and tonal context, especially for the more complex Beethoven chord progression, could also be useful for providing more insight into the results of the parser (As shown in Table 3).

Table 3. Full chord progression used in visual analysis, shown as pitch-class data with timing metadata.

[E#, 4, G#, 4, B, 4, 240, 25]	[B, 3, E#, 4, G#, 4, 240, 25]	[A#, 3, D#, 4, F##, 4, 240, 26]
[A#, 3, D#, 4, F##, 4, 240, 26]	[B, 3, D#, 4, G#, 4, 240, 27]	[D#, 4, F##, 4, A#, 4, 240, 27]
[D#, 4, G#, 4, B, 4, 240, 28]	[D#, 4, F##, 4, A#, 4, 240, 28]	[D#, 4, G#, 4, B, 4, 240, 29]
[F#, 4, A#, 4, C#, 5, 240, 29]	[F#, 4, B, 4, D#, 5, 240, 30]	[F#, 4, A#, 4, C#, 5, 240, 30]
[F#, 4, B, 4, D#, 5, 240, 31]	[A#, 4, C##, 5, E#, 5, 240, 31]	[A#, 4, D#, 5, F#, 5, 240, 32]
[A#, 4, C##, 5, E#, 5, 240, 32]	[A#, 4, D#, 5, F#, 5, 240, 33]	[C#, 5, E#, 5, G#, 5, 240, 33]
[C#, 5, F#, 5, A#, 5, 240, 34]	[D#, 5, F#, 5, B, 5, 240, 34]	[B, 4, F#, 5, G#, 5, 240, 35]
[C#, 5, F#, 5, A#, 5, 240, 35]	[D#, 5, F#, 5, B, 5, 240, 36]	[F#, 5, A#, 5, C#, 6, 240, 36]
[F#, 5, B, 5, D#, 6, 240, 37]	[G#, 5, B, 5, E, 6, 240, 37]	[E, 5, B, 5, C#, 6, 240, 38]
[F#, 5, B, 5, D#, 6, 240, 38]	[G#, 5, B, 5, E, 6, 240, 39]	[B, 5, D#, 6, F#, 6, 240, 39]
[F#, 5, B, 5, D#, 6, 240, 40]	[G#, 5, B, 5, E, 6, 240, 40]	[B, 5, D#, 6, F#, 6, 240, 41]

4.13. First Version

The parser output the chords like this, each element below consists of [note1, octave1, note2, octave2, note3, octave3, duration, bar number]

These chords are then converted into MIDI format for ease of manipulation. The table of the MIDI notes is seen in Section 5.13 (As shown in Table 4).

Table 4. Raw triad data extracted before MIDI conversion.

[65, 68, 71]	[59, 65, 68]	[58, 63, 67]
[58, 63, 67]	[59, 63, 68]	[63, 67, 70]
[63, 68, 71]	[63, 67, 70]	[63, 68, 71]
[66, 70, 73]	[66, 71, 75]	[66, 70, 73]
[66, 71, 75]	[70, 74, 77]	[70, 75, 78]
[70, 74, 77]	[70, 75, 78]	[73, 77, 80]
[73, 78, 82]	[75, 78, 83]	[71, 78, 80]
[73, 78, 82]	[75, 78, 83]	[78, 82, 85]
[78, 83, 87]	[80, 83, 88]	[76, 83, 85]
[78, 83, 87]	[80, 83, 88]	[83, 87, 90]
[78, 83, 87]	[80, 83, 88]	[83, 87, 90]
[83, 88, 92]	[84, 90, 93]	[88, 91, 94]

Prior to the 3D visualization of the chords, a 2D visualization is created. This maps the sequence of triads that were identified from measures 25-42 of the current piece to a discrete coordinate space where the horizontal axis represents the index of the lowest pitch in the triad and the vertical axis represents the index of the middle pitch.

The color mapping represents the pitch class of the highest note in each triad of the highest note in the triad. This is done using a hue-based mapping in which pitch classes are evenly distributed across the color wheel. There are twelve equally distributed divisions across the color wheel. For example, C corresponds to hue 0, G# corresponds to hue 0.75, and so on. The color mapping represents the pitch class of the highest note in each triad of the highest pitch. This provides an octave-dependent visual cue in which lower pitches will appear darker while higher pitches will appear lighter. This allows for chords to be visually distinguished even if they share the same pitch class structure.

In the resulting 2D visualization, there is a horizontal clustering effect in which multiple triads share the same lowest note. This indicates harmonic stasis in the bass. Similarly, there is a vertical clustering effect in which multiple triads share the same middle note. This indicates stepwise motion in the outer voices while keeping the inner voice steady on a single pitch. The diagonal lines in the resulting 2D visualization indicate a chord progression in which the entire triad moves in a transpositional pattern.

The actual measure numbers are also provided as an overlay adjacent to each plotted point, thus facilitating the direct temporal interpretation of harmonic events. As such, it becomes quite simple to track the chord progression measure by measure, as well as identify instances where there may be repetition or returning to previous harmonic events. For instance, repeated positions in the visualization, where the hue and brightness levels are the same, can indicate repeated harmonic events at the same pitch levels, while repeated positions with varying brightness levels can indicate octave displacement of the same harmonic content.

The legend provided indicates the relationship between hue and pitch class, thus providing the viewer with a key to understand the specific color assignments. The placement of the pitch class legend outside the main plotting area also facilitates the visualization, as it allows for maximum space to be provided in the visualization itself. The provision of a grid also facilitates the visualization, particularly in identifying the exact pitch class positions in the indices, as well as the vertical and horizontal distances between these points.

From an analytical point of view, this visualization of the data, represented in this 2D form, can function as an important tool in the analysis of harmonic function as well as voice leading without necessarily having to deal with traditional notation systems. The low and middle note indices function as stable anchors, thus facilitating greater interpretability with regard to the visualization provided. As such, this visualization can function as an important bridge between discrete pitch class data and perception in harmonic progression (As shown in Figure 3 and Figure 4).

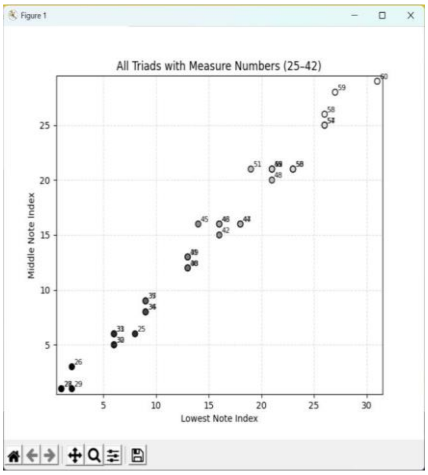

Figure 3. 2D pitch-position plot with grayscale brightness encoding octave.

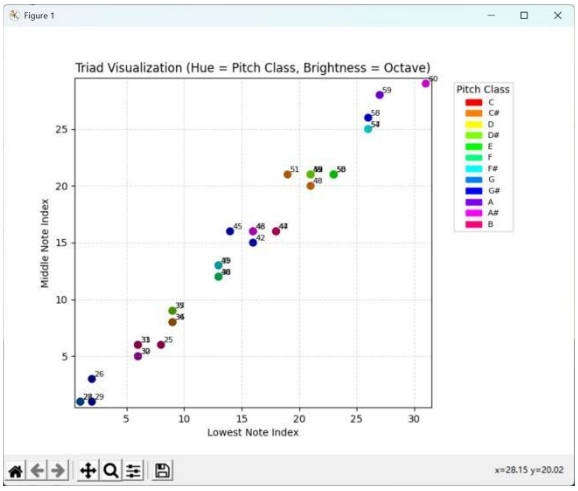

Figure 4. 2D pitch-position plot with hue encoding pitch class and brightness encoding octave.

5. Conclusion

The analysis of chord progressions has long been a central topic in music theory, particularly within the neo-Riemannian framework, where chord transformations offer valuable insights into harmonic structure and motion. Traditional methods of chord analysis rely on manual reasoning and symbolic notation, yet these approaches often fail to handle complex or irregular chord progressions, especially when dealing with computer-generated or non-idealized sequences that deviate from conventional pitch norms. As a result, the challenge of designing an effective computational model that can automatically process these chord progressions and reveal their underlying structure has become a significant issue in music theory research.

In this study, we sought to address the challenges that come with the analysis of chord progressions using the application of computational methods by developing a parser for chords within the neo-Riemannian theory. In this case, we considered the Parallel (P), Leittonwechsel (L), and Relative (R) transformations. These transformations form the major concepts in the Tonnetz model. In this case, we had to process the musical data in a symbolic form and obtain the pitch class sets and represent the sets in geometric tonal space. The findings from this study show that the irregularities that come with the progression of the chords, even those that are generated by the algorithm, do not in any way hinder the application of the geometric model in the analysis of the transformational patterns that come with the musical piece. This shows that the application of the computational model that we proposed in this study is reliable and operational in the analysis of the irregularities that come with the progression of the chords. Moreover, this study shows that the application of the computational model in the analysis of musical pieces offers a more intuitive perspective compared to the conventional approaches. For instance, the application of the model in the analysis of the chord progressions that come with the compositions of Beethoven showed the potential for the application of the transformational network in the analysis of the relationships that come with the musical piece. This demonstrated the potential that the model had in the creation of a visualization tool that would be applicable in the teaching of music, thus enhancing the understanding of the concepts that were underlying within the musical piece. Furthermore, the model was able to demonstrate its potential in the identification of geometric patterns within the analysis of the randomly generated chord sequences. This demonstrated the potential that the model had in the analysis of more complex musical pieces. As one looks to the future, it is evident that although the study provides new avenues of thought regarding the analysis of chord progressions, there is always room for expansion. First, it is evident that although the parser was able to perform adequately with classical pieces of music, such as Beethoven's works, its adaptability would need to be explored. Secondly, future research would allow for the parser's adaptability to be expanded upon by adding additional layers of information that would allow it to delve deeper into the analysis of music. By taking these additional layers of information into consideration, it would allow the parser to extend beyond the analysis of chord structure and delve deeper into the dynamic evolution of music. This would allow it to provide a comprehensive analysis of more complex musical pieces.

Acknowledgments: The author wishes to extend their sincerest appreciation to Professor Anthony Della Pella. The professor was very instrumental in the conceptual framework and analysis that is presented within this study. The author wishes to extend their appreciation to Yvonne. The assistance that she provided in the editing and refining of the written presentation ensured that the manuscript presented within this study was precise and coherent. In addition to the direct influences, the author would like to extend their appreciation to the academic environment that allowed for interdisciplinary research, bringing the concepts of computation, visualization, and theory into one paper. The development of this paper was as much an exploration of the methods as it was the music, and the author would like to extend their appreciation to those who helped that exploration with patience, expertise, and encouragement.

Funding: The author did not receive any specific funding for the research of this paper or the writing of this article.

Conflicts of Interest: The author declares that there are no conflicts of interest regarding the publication of this article.

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